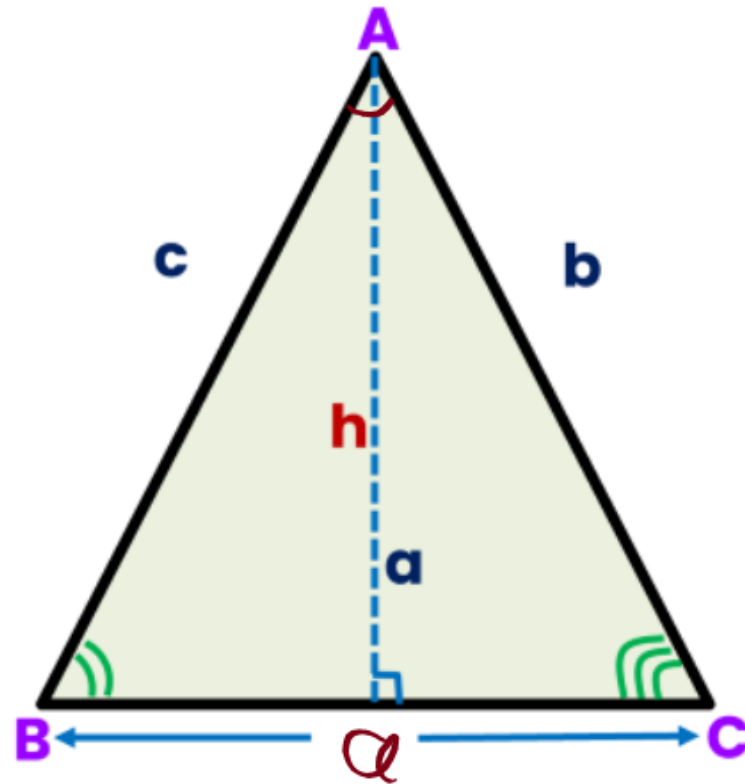


Sine Rule



$$\diamond \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \text{ (constant)}$$

$$a = K \sin A,$$

$$b = K \sin B,$$

$$c = K \sin C,$$

(Side)

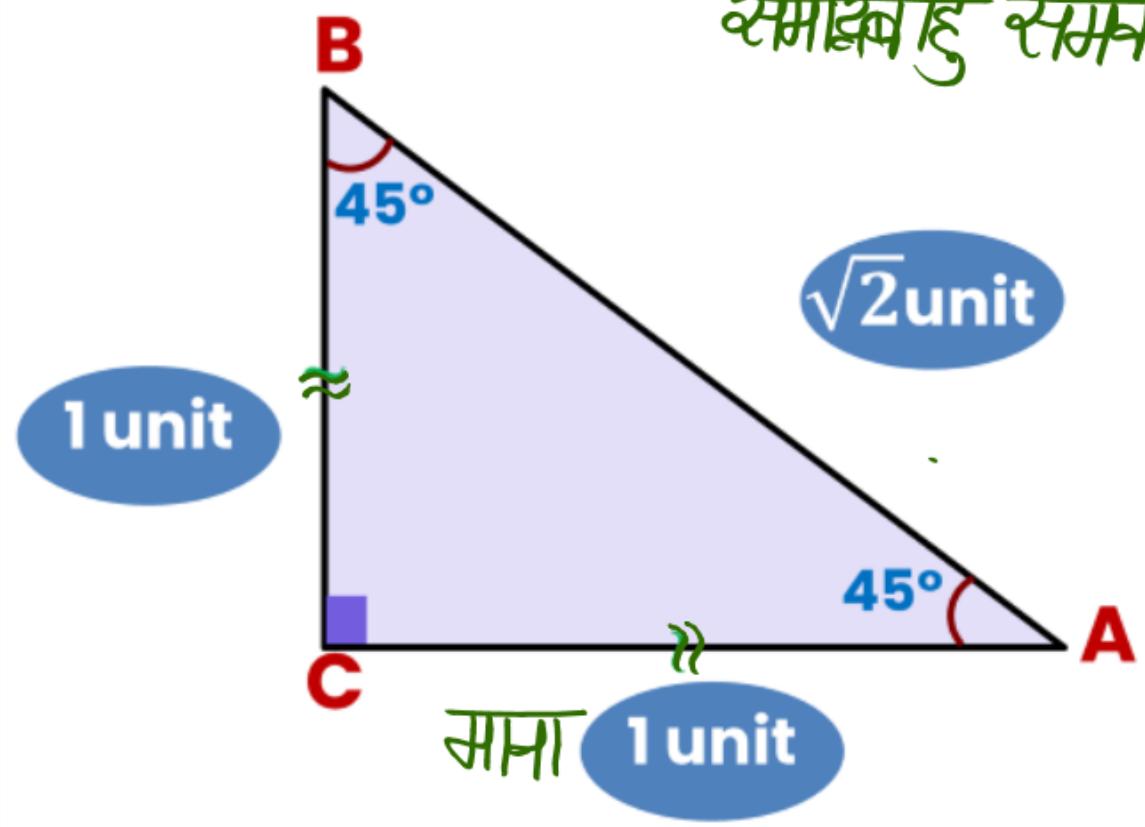
$$a : b : c = \sin A : \sin B : \sin C$$



special $\triangle \Rightarrow$

$45^\circ - 45^\circ - 90^\circ \rightarrow$ Isosceles Right Angle Triangle.

समबाहु समकोण $\triangle$



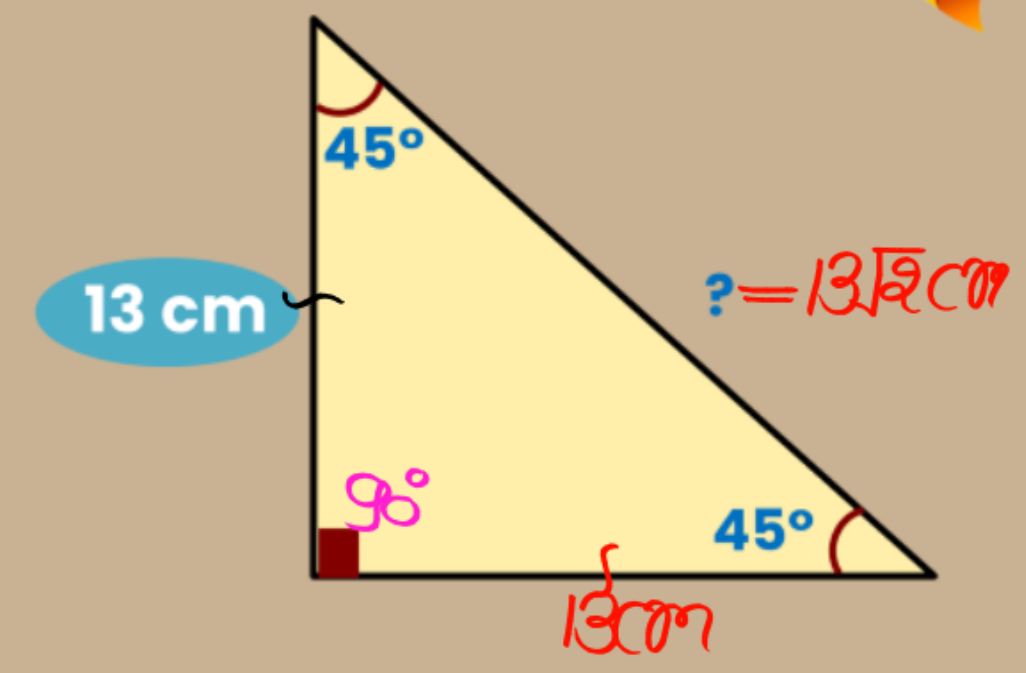
45°
 $\downarrow$
 1

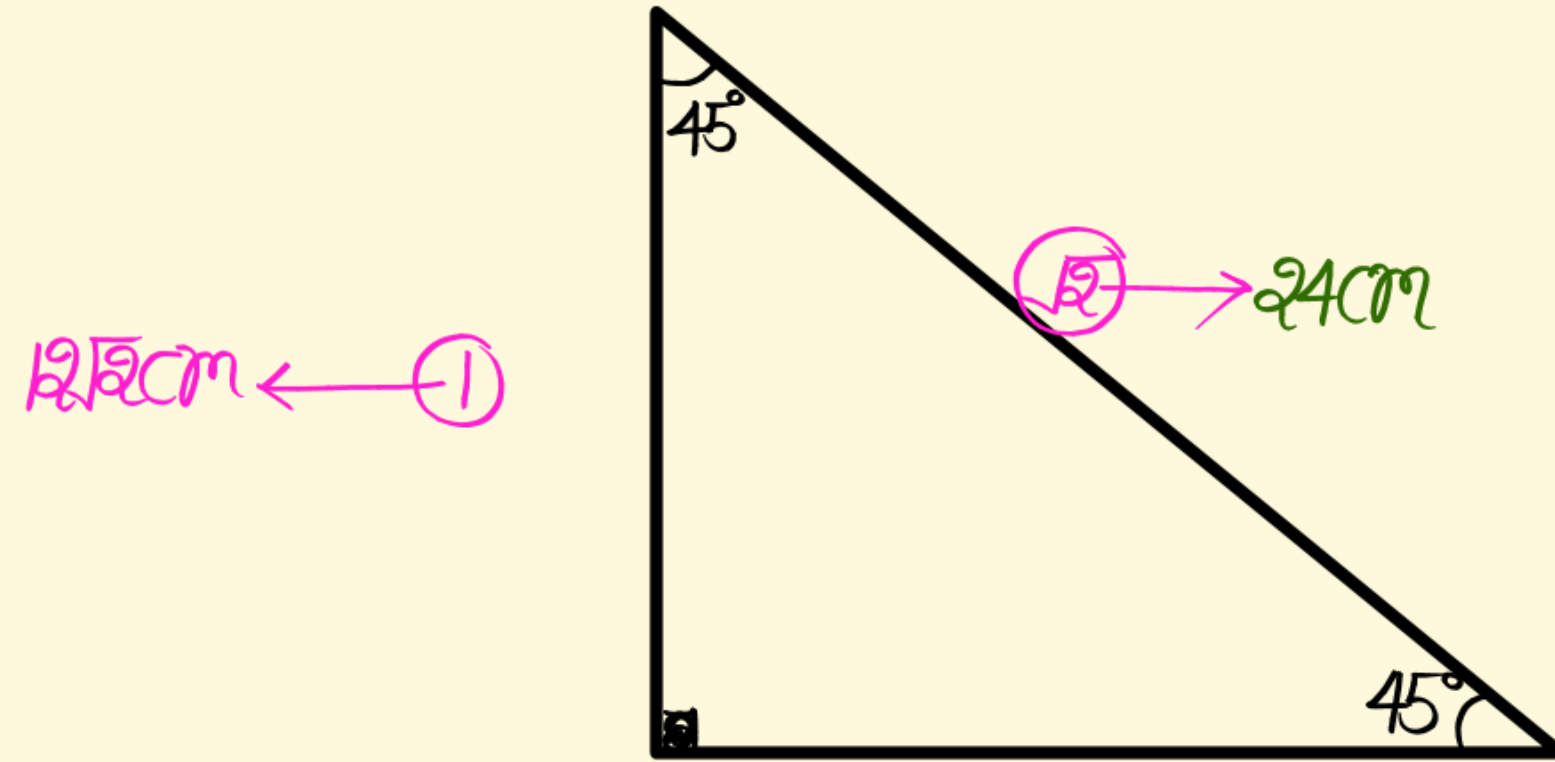
45°
 $\downarrow$
 1

90°
 $\downarrow$
 $\sqrt{2}$

$\rightarrow$ Side

Example:

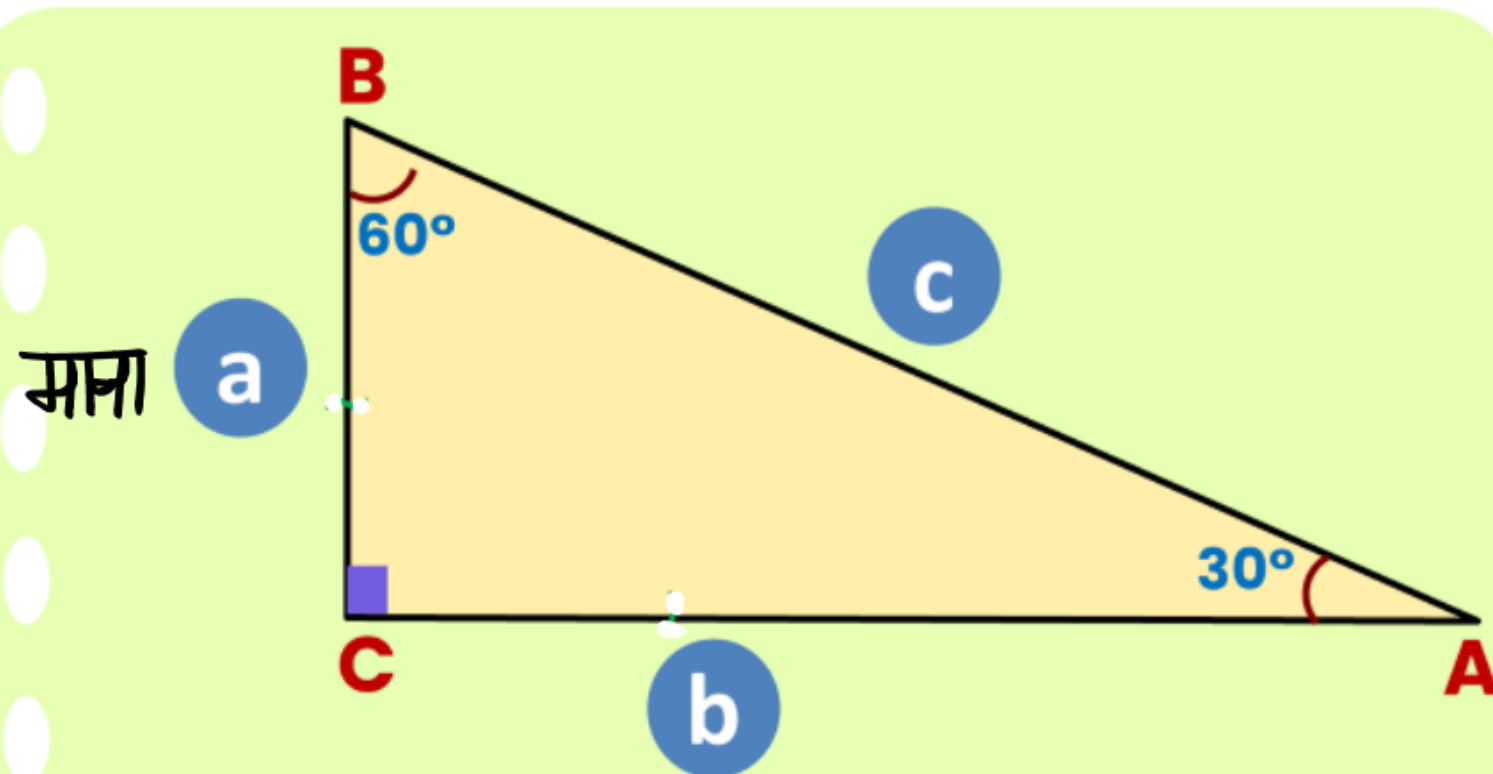




$$\frac{24}{\sqrt{2}} = \frac{12 \times \cancel{2}}{\cancel{\sqrt{2}}}$$



#



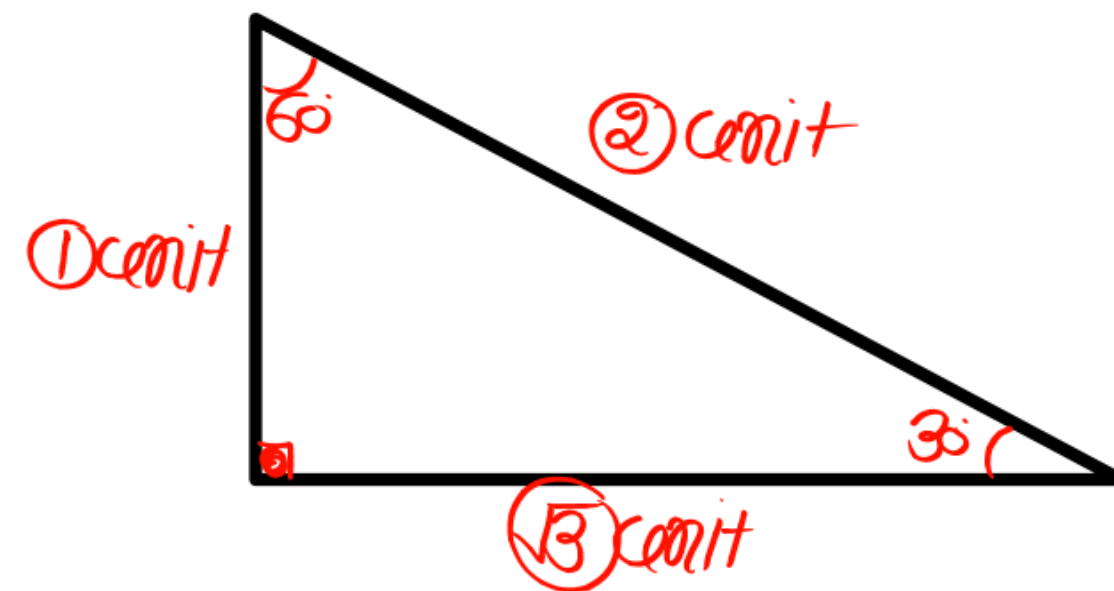
माना

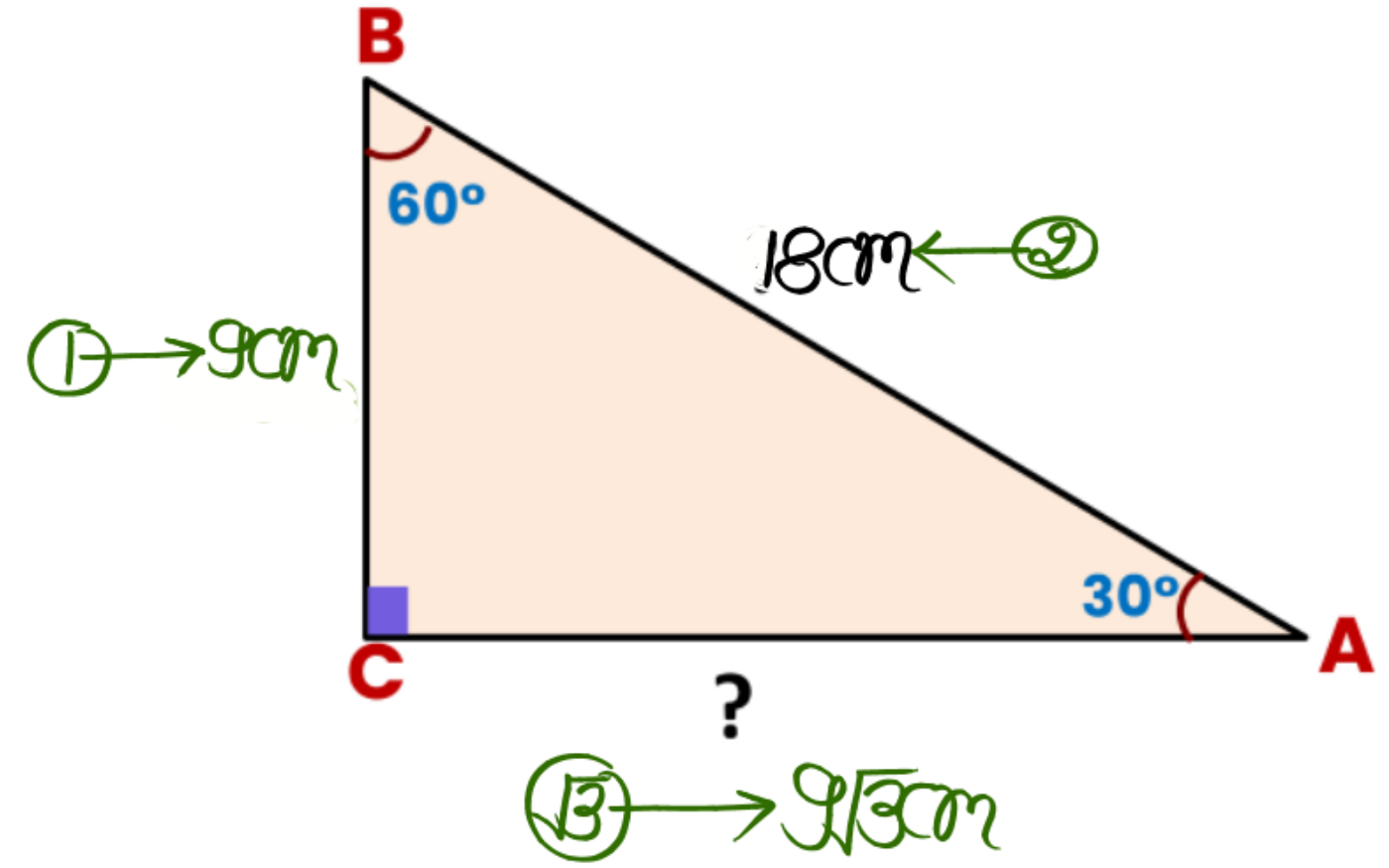
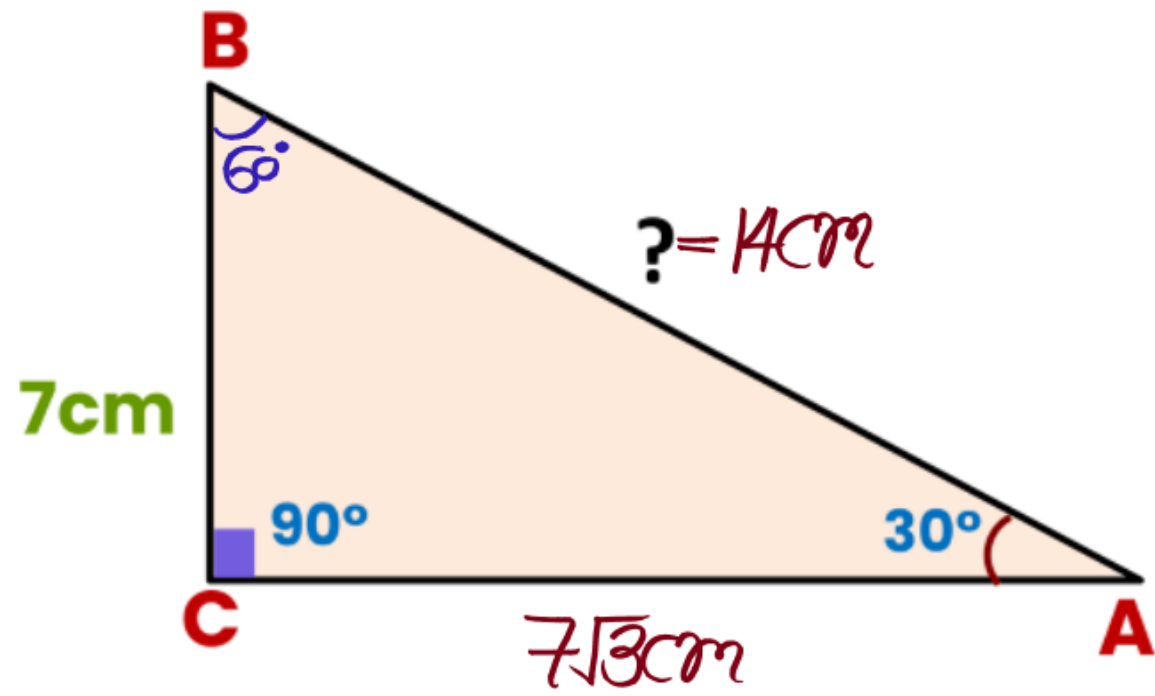
$$a : b : c = \sin 30^\circ : \sin 60^\circ : \sin 90^\circ$$

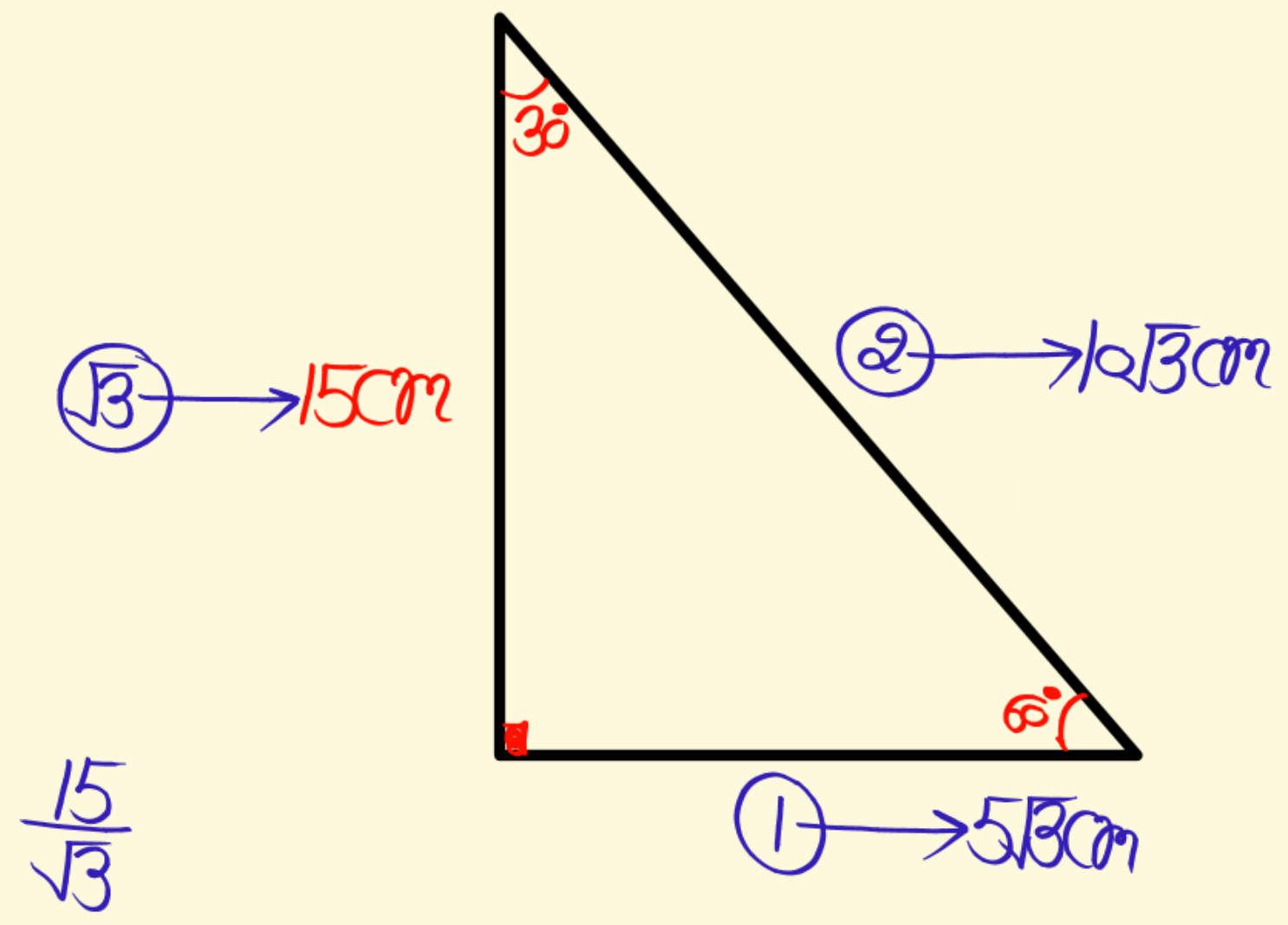
$$= \left(\frac{1}{2} : \frac{\sqrt{3}}{2} : 1 \right) \times 2$$

$$= 1 : \sqrt{3} : 2$$

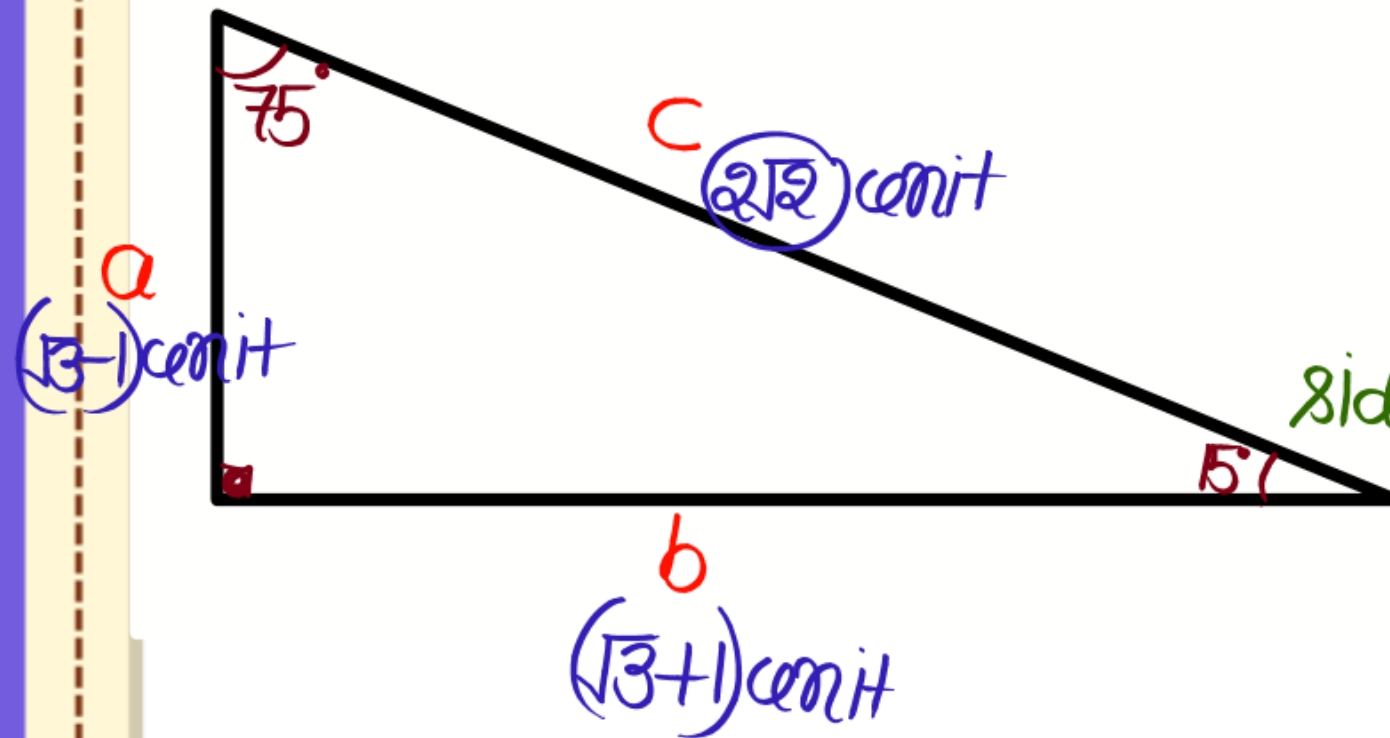
30° 60° 90°
sides $\rightarrow 1 : \sqrt{3} : 2$







$15^\circ - 75^\circ - 90^\circ \Rightarrow$



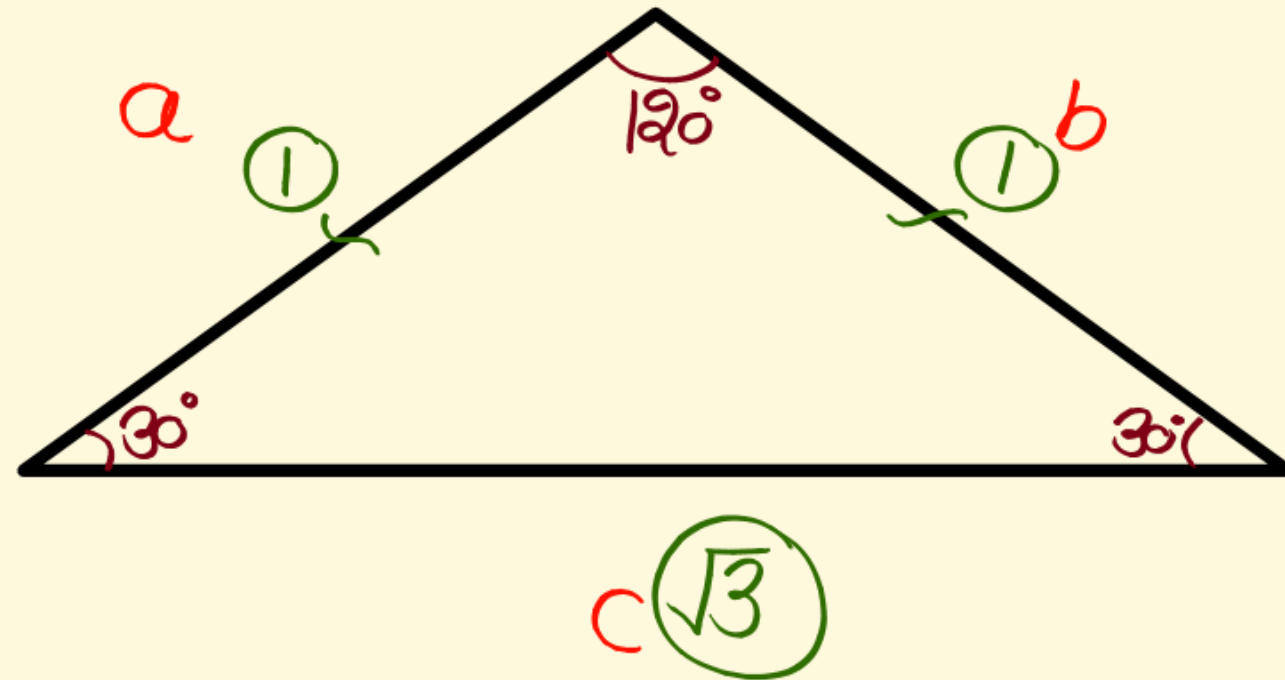
$a : b : c$

$= \sin 15^\circ : \sin 75^\circ : \sin 90^\circ$

$$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}} : 1 \right) \times 2\sqrt{2}$$

sides $= \sqrt{3}-1 : \sqrt{3}+1 : 2\sqrt{2}$

30°-30°-120° ⇒



$$\left. \begin{aligned} \sin 120^\circ &= \frac{\sqrt{3}}{2} \\ \cos 120^\circ &= -\frac{1}{2} \end{aligned} \right\}$$

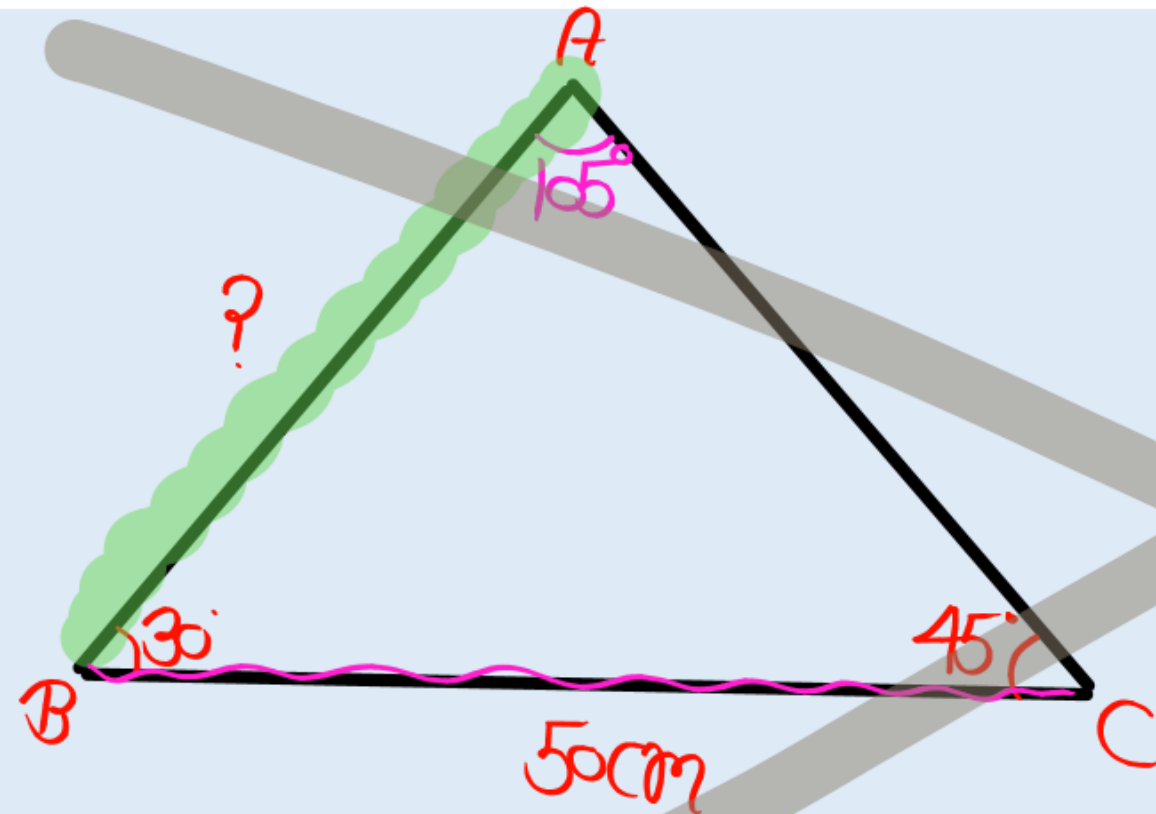
$$\begin{array}{ccc} a & b & c \\ \sin 30^\circ & \sin 30^\circ & \sin 120^\circ \\ \frac{1}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{array}$$

sides → 1 : 1 : $\sqrt{3}$

13. In a triangle ABC, $\angle B=30^\circ$ and $\angle C=45^\circ$. If $BC=50\text{cm}$ then find the length of AB?
किसी त्रिभुज ABC में $\angle B=30^\circ$ और $\angle C=45^\circ$ है। यदि $BC=50\text{cm}$ है तो AB की लंबाई ज्ञात करें।

[A] $\frac{50}{\sqrt{3}+1}$
[C] $\frac{100}{(\sqrt{3}-1)}$

[B] $50(\sqrt{3}-1)$
[D] $100(\sqrt{3}-1)$



$$\frac{AB}{50\text{cm}} = \frac{\sin 45^\circ}{\sin 105^\circ}$$

$$AB = 50\text{cm} \times \frac{1}{\frac{\sqrt{3}+1}{2}}$$

$$AB = \frac{100}{(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$AB = \frac{50}{1} \times (\sqrt{3}-1)$$

$$\sin 105^\circ = \sin 75^\circ = \frac{\sqrt{3}+1}{2}$$

14. If the angles of a triangle are 30° and 45° and the included side is $(\sqrt{108} + 6)$ cm, then what is the area of the triangle?

यदि किसी त्रिभुज के कोण 30° और 45° हैं और उसकी सम्मिलित भुजा $(\sqrt{108} + 6)$ सेमी है, तो त्रिभुज का क्षेत्रफल क्या है?

[A] $18(\sqrt{3} + 1) \text{ cm}^2$

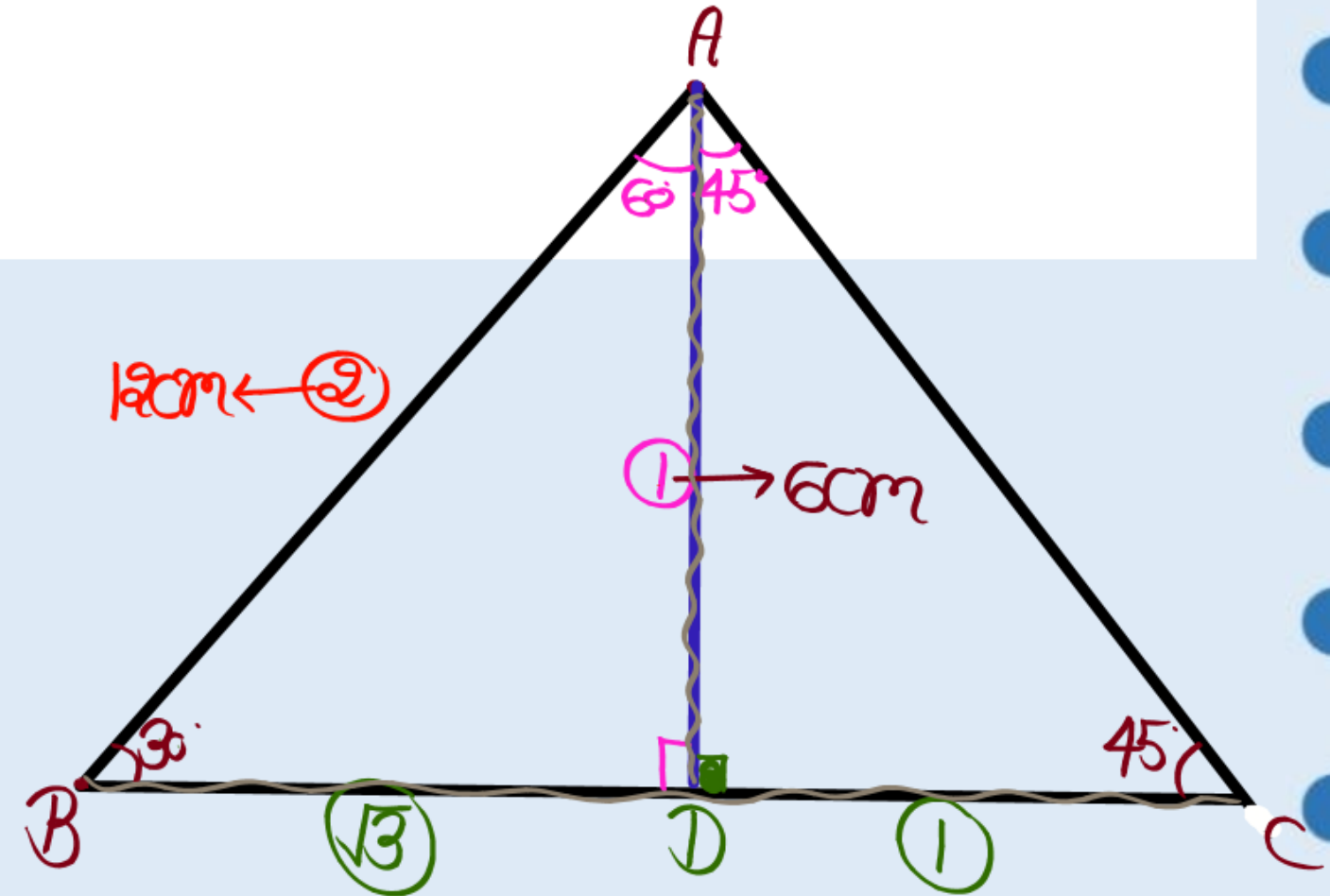
[B] $(15\sqrt{3} + 18) \text{ cm}^2$

[C] $12(\sqrt{3} + 1) \text{ cm}^2$

[D] $24(\sqrt{3} + 1) \text{ cm}^2$

$\frac{1}{2} \times 6(\sqrt{3} + 1) \times 6 \text{ cm}^2$

$\sqrt{36 \times 3} + 6$
 $6\sqrt{3} + 6$
 $6(\sqrt{3} + 1)$



$(\sqrt{3} + 1) \times 6 \rightarrow 6(\sqrt{3} + 1) \text{ cm}$

15. In a triangle ABC, $\angle A = 30^\circ$, $AB = 7$ cm and $AC = 12$ cm. What is the area of the triangle ABC?

एक त्रिभुज ABC में, $\angle A = 30^\circ$, $AB = 7$ सेमी और $AC = 12$ सेमी है। त्रिभुज ABC का क्षेत्रफल क्या है?

(UPSC CDS - II 2025)

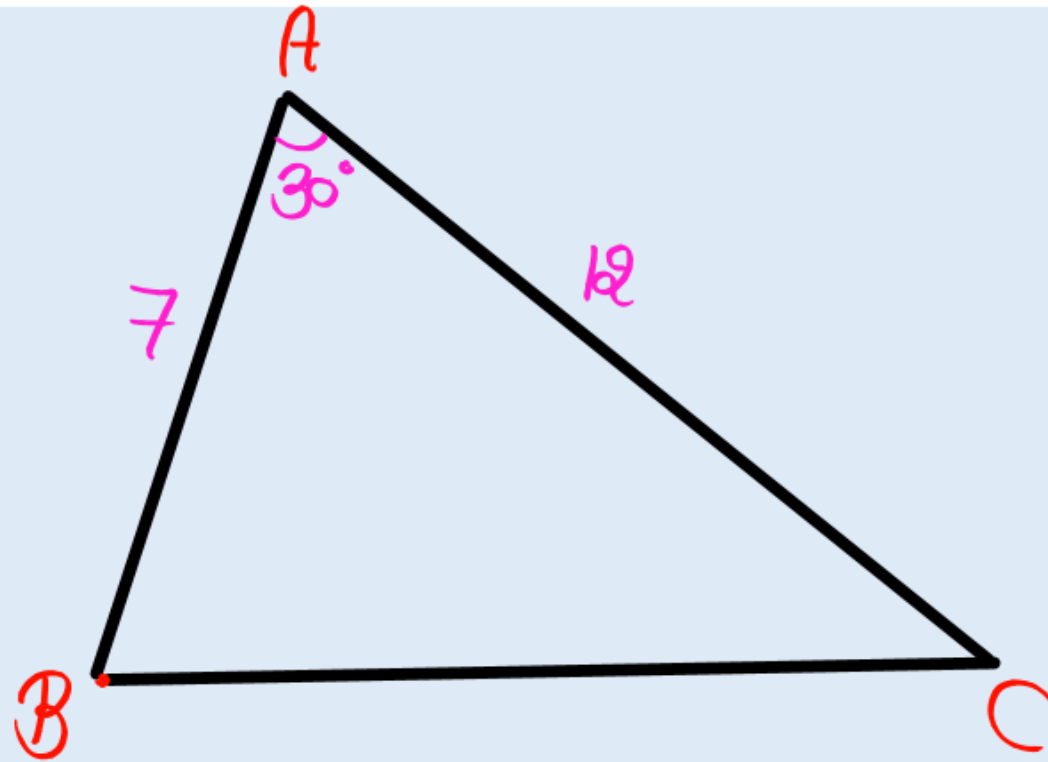
[A] 21 cm^2

[B] $21\sqrt{3} \text{ cm}^2$

[C] 42 cm^2

[D] $42\sqrt{3} \text{ cm}^2$

$$= \frac{1}{2} \times 7 \times 12 \times \frac{1}{2} = 21 \text{ cm}^2$$



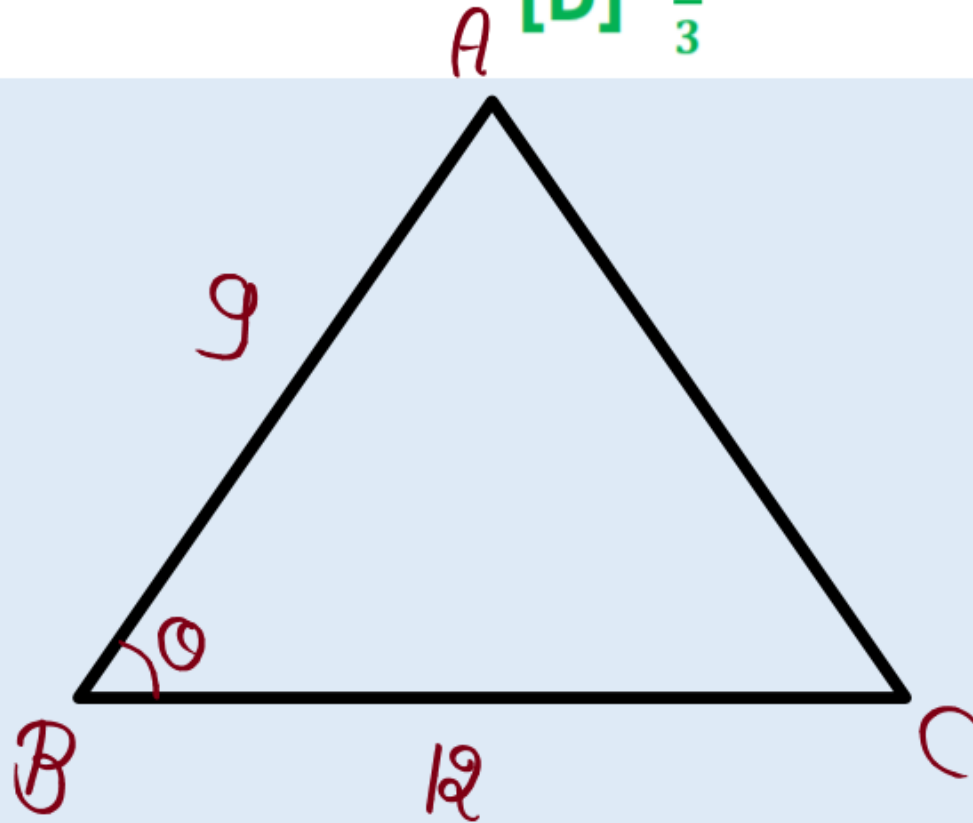
16. Let ABC be a triangle with area 36 square cm. if $AB = 9$ cm, $BC = 12$ cm and $\angle ABC = \theta$, then what is $\cos\theta$ equal to?

माना कि ABC एक त्रिभुज है जिसका क्षेत्रफल 36 वर्ग सेमी है। यदि $AB = 9$ सेमी, $BC = 12$ सेमी और $\angle ABC = \theta$ है, तो $\cos\theta$ किसके बराबर है?

(UPSC CDS-2 2024)

[A] $\frac{\sqrt{5}}{3}$
[C] $\frac{1}{3}$

[B] $\frac{\sqrt{5}}{4}$
[D] $\frac{2}{3}$



~~$\frac{1}{2} \times 9 \times 12 \times \sin\theta = 36$~~

$\sin\theta = \frac{2}{3}$

$\cos\theta = \sqrt{1 - \sin^2\theta}$
 $= \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

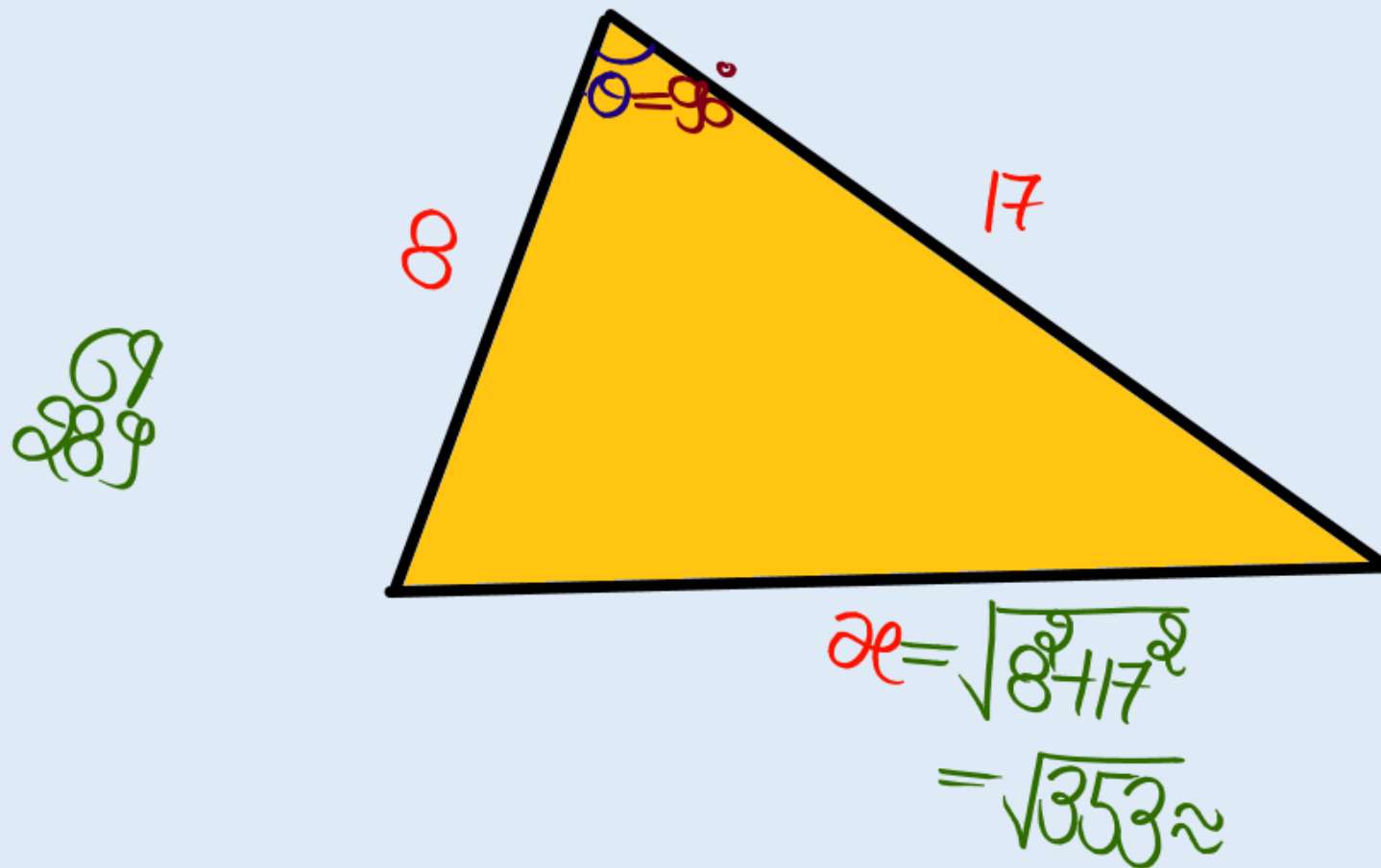
17. If three sides of a triangle are 8cm, 17cm and x cm. for what value of x area of triangle is maximum?

एक त्रिभुज की तीन भुजायें 8 सेमी, 17 सेमी और x सेमी है, x के किस मान के लिए त्रिभुज का क्षेत्रफल अधिकतम होगा?

[A] ~~15cm~~
[C] 18.79cm

[B] ~~17cm~~
[D] ~~23.16cm~~

$$\text{max Area} = \frac{1}{2} \times 8 \times 17 \times \sin \theta_{\text{max}} \\ = 68 \text{ cm}^2$$



$$\sin \theta_{\text{max}} = 1 \quad \text{at } \theta = 90^\circ$$

18. One side of a triangle has length 8 and a second side has length 5. Which of the following could be the area of the triangle?

एक त्रिभुज के एक भुजा की लंबाई 8 और दूसरी भुजा की लंबाई 5 है. निम्न में से कौन सा त्रिभुज का क्षेत्रफल हो सकता है?

I. ~~24~~

[A] I only

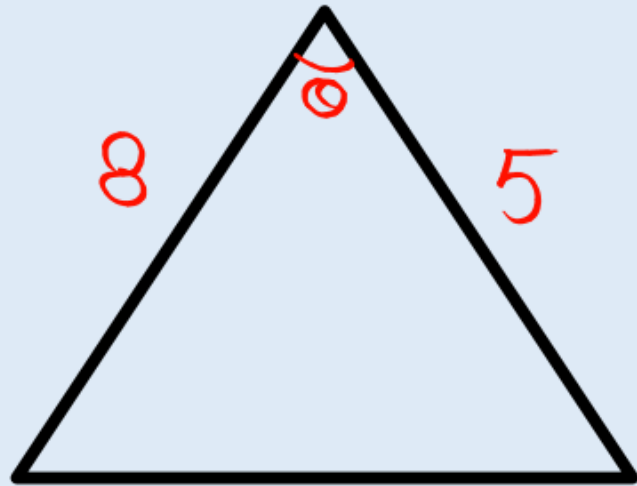
[C] III only

II. 20 ✓

[B] II only

[D] II and III only

III. 5 ✓



$$\text{Area} = \frac{1}{2} \times 8 \times 5 = 20$$

19. In a triangle ABC, AB=8cm, AC=12cm, AD is the angle bisector of $\angle BAC$. If $\angle BAC = 60^\circ$. What is the length of AD?

त्रिभुज ABC में, AD, $\angle BAC$ का कोण समद्विभाजक है। $\angle BAC = 60^\circ$, AB=8cm और AC=12cm, AD की लंबाई क्या है?

[A] $\frac{32\sqrt{3}}{5}$ cm

[B] $\frac{20\sqrt{3}}{3}$ cm

[C] $\frac{25\sqrt{3}}{7}$ cm

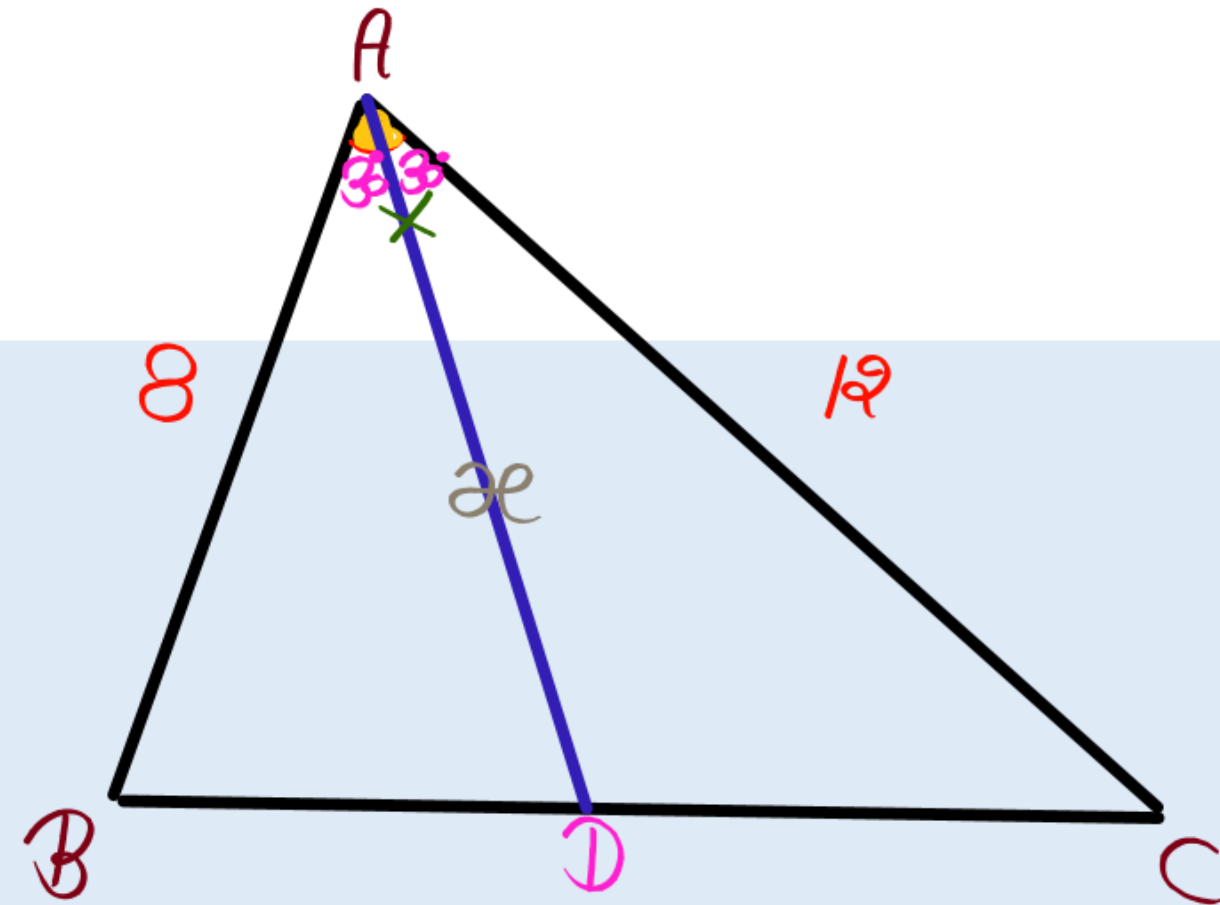
[D] $\frac{24\sqrt{3}}{5}$ cm

area $[\triangle ABC = \triangle ADB + \triangle ADC]$

$$\frac{1}{2} \times 8 \times 12 \times \frac{\sqrt{3}}{2} = \frac{1}{2} \times 8 \times x \times \frac{1}{2} + \frac{1}{2} \times 12 \times x \times \frac{1}{2}$$

$$\frac{24}{5}\sqrt{3} = x$$

$$\frac{24\sqrt{3}}{5} = x$$



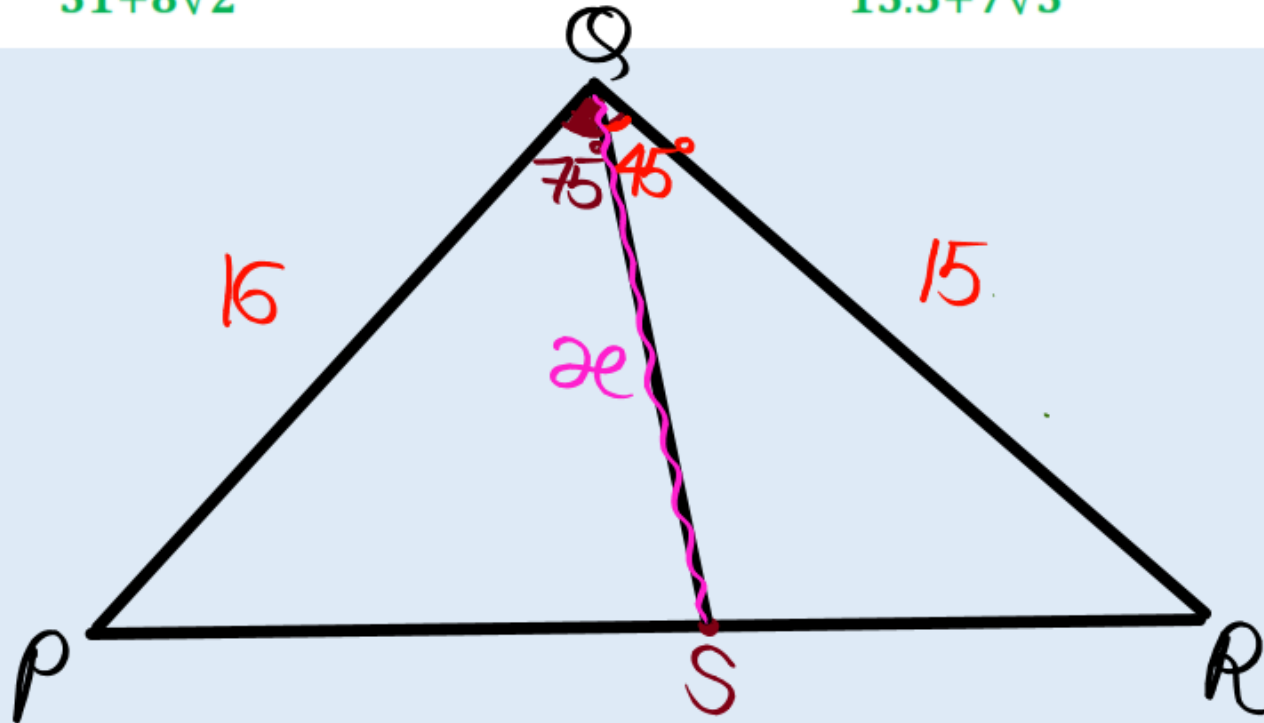
20. In a ΔPQR , $\angle PQR = 120^\circ$. S is a point on side PR such that $\angle PQS = 75^\circ$, if $PQ = 16\text{cm}$ and $QR = 15\text{cm}$. then find the length of QS?

त्रिभुज PQR में $\angle PQR = 120^\circ$. भुजा PR पर एक बिंदु S इस प्रकार स्थित है कि $\angle PQS = 75^\circ$ और $PQ = 16\text{cm}$, $QR = 15\text{cm}$ तब QS की लंबाई होगी?

[A] $\frac{120\sqrt{6}}{23+8\sqrt{3}}$
 [C] $\frac{240\sqrt{2}}{31+8\sqrt{2}}$

[B] $\frac{124\sqrt{6}}{27+11\sqrt{3}}$
 [D] $\frac{120\sqrt{3}}{15.5+7\sqrt{3}}$

$$\cancel{16} \times 15 \times \frac{\sqrt{3}}{2} = \cancel{16} \times \cancel{x} \times \frac{(\sqrt{3}+1)}{(\cancel{2}\sqrt{2})} + 15 \times \cancel{x} \times \frac{1}{\sqrt{2}}$$



$$15\sqrt{3} = \frac{x(\sqrt{3}+1)}{\sqrt{2}}$$

$$\frac{15\sqrt{6}}{\sqrt{3}+1} = x$$